# Technical Notes

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## Prediction of Transverse Asymmetries in MHD Ducts with Zero Net Hall Current

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### I. Introduction

IN this Note a new class of fluid-electrical asymmetries in MHD generator channel flow are predicted. Asymmetrical behavior near the two electrode walls of an MHD channel has been confirmed both experimentally and theoretically for devices in which there exists a net axial current density of significant magnitude. Demetriades,[1] for example, investigated the momentum transfer to plasmas by Lorentz forces in order to predict the changes in direction and magnitude of the momentum of a stream of plasma in a crossed-field accelerator. In the AEDC Hirho accelerator[2] measurements of the dynamic pressure across the channel revealed that the peak of these profiles occurred a considerable distance off the centerline. Theoretical studies have also been reported which illustrate that the boundary layer on one wall or the other (depending on the direction of the axial Lorentz force) will have a greater tendency to separate.[3] Such transverse nonuniformities in the gasdynamic distributions have been shown to have great influence on the current distributions and voltage drops in the channel and, consequently, on the power output and efficiency of the generator.[4]

These nonuniformities arise because of the axial current density $J_x$, which in turn is due to the Hall effect, finite segmentation, and nonuniformity of plasma properties over the electrode segment. It is the purpose of this Note to: 1) illustrate that the existence of interelectrode asymmetries is not confined solely to generators in which there exists a nonzero net axial current, but rather they are induced even in the case of the ideal Faraday generator, and 2) reveal the impact of these asymmetries upon the generator and diffuser flow. In generators of this type the net axial current in the cross plane is identically zero but at any given point in the plane the local Hall current density is in general nonzero.

### II. Model

In the formulation and solution of the MHD equations, two overlapping subproblems are distinguished. They are: 1) the electrical subproblem, which consists of the solution for the current densities and electric fields assuming specified values of the gasdynamic variables; and 2) the gasdynamic subproblem, which consists of the solution for all of the fluid variables and the plasma state throughout the channel assuming specified values for the current densities and electric fields.

In the general situation, the equations governing the electrical subproblem consist of the steady-state Maxwell's equations, the generalized Ohm's law,[5] and the electron species and energy conservation equations. The equations governing the gasdynamic subproblem are derived from the steady state, compressible, turbulent Navier-Stokes equations including MHD body-force and Joule heating terms. A mathematical description of the general computational model (the "STD Q3D" flow model)[6-9] will not be repeated here. Particular assumptions employed in application of the above general model to the present work are as follows: 1) the neglect of finite-electrode segmentation effects, 2) the assumption of equilibrium heavy species concentrations in the boundary-layer regions (electrons, however, are treated with finite-rate effects for both concentration and electron energy), and 3) the assumption of an equilibrium-turbulence state in the boundary-layer regions in which the turbulent shear stress is given in terms of the velocity gradient $\partial U/\partial y$ by

$$\tau \equiv \epsilon_t \frac{\partial U}{\partial y}, \qquad \epsilon_t = \rho (a\delta)^2 \left| \frac{\partial U}{\partial y} \right|$$

where the dissipation length $a\delta$ is given by a modified Van Driest formulation[10] and is discussed fully in Ref. 7. The boundary conditions imposed on the above equations are derived from a modified law of the wall for rough walls and are discussed in Ref. 11.

Fully nonequilibrium turbulence models[6] have been employed and compared extremely favorably with data in non-MHD flows.[12,13] We expect that the equilibrium turbulence version used in the present work is adequate for the documentation of the asymmetric MHD flow effects whose existence and character are the subject of this Note and whose fundamental nature and mechanism do not turn directly on the question of turbulence equilibrium or nonequilibrium. As described in Ref. 9, coupled solutions for the interelectrode direction and intersidewall direction are obtained simultaneously in a quasi-three-dimensional description which correctly distinguishes the differences in primary flow velocity distribution over sidewalls and electrode walls but which can only approximately account for secondary flow effects.

### III. Results

The foregoing version of the STD Q3D flow model has been studied for inlet conditions, geometry, and electrical loading typifying a large, rectangular cross-section, high MHD interaction, subsonic, Faraday generator/diffuser combination (Table 1). Many of the results of this calculation are reported in Ref. 14. We now comment on an aspect of the calculation left unmentioned in the above reference.

Figure 1 is a schematic of typical Hall current densities in the interelectrode direction considering both zero and nonzero net Hall current. Also depicted in Fig. 1 are typical pressure distributions which would be expected to result from the transverse Lorentz forces. Indeed, it is clear that even in a situation of zero net Hall current, there is a significant asymmetry in the pressure distribution from anode to cathode. While the pressure at the surface is the same on both

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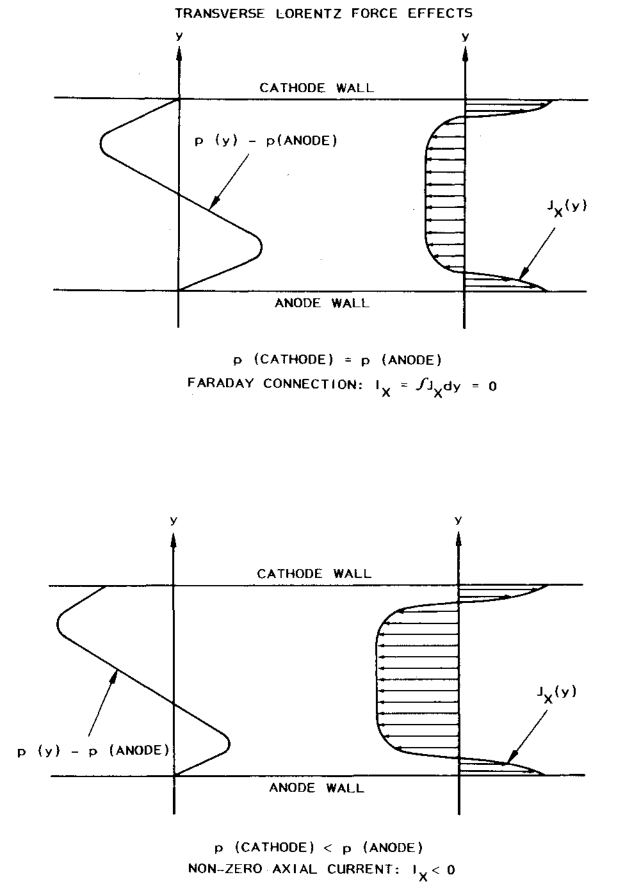


**Fig. 1 Effect of Hall current on pressure distribution between electrodes.**

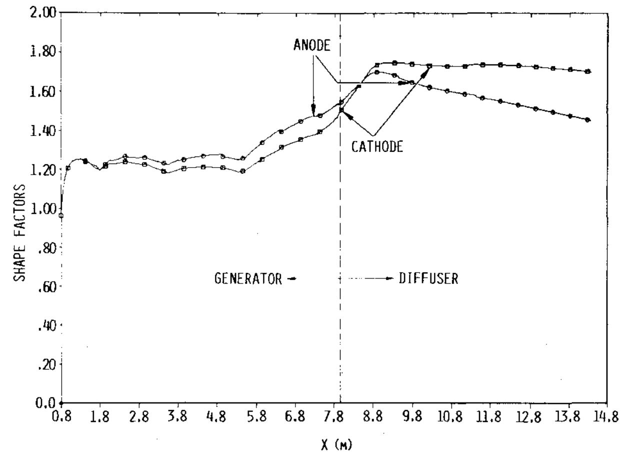


**Fig. 2 Distribution of anode and cathode shape factors for a typical high interaction MHD generator and diffuser.**

**Table 1 MHD channel flow conditions**

| | Generator inlet | Generator exit | Diffuser exit |
|---|---|---|---|
| Mass flow rate, kg/s | 45 | 45 | 45 |
| Centerline velocity, m/s | 890 | 557 | 359 |
| Centerline temperature, K | 2815 | 2501 | 2467 |
| Centerline pressure, atm | 2.897 | 0.914 | 1.0 |
| Average electrode wall boundary-layer thickness, m | .012 | .241 | .492 |
| Centerline Mach no. | 0.934 | 0.630 | 0.409 |
| Cross-sectional area, $m^2$ | 0.140 | 0.707 | 1.242 |

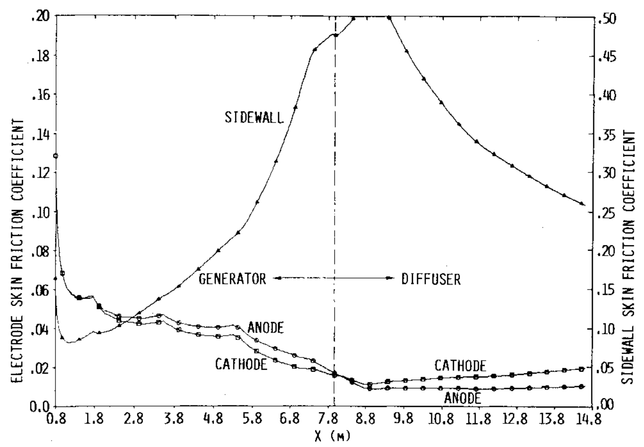


**Fig. 3 Skin-friction distributions on electrode and sidewalls for a typical high interaction MHD generator and diffuser.**

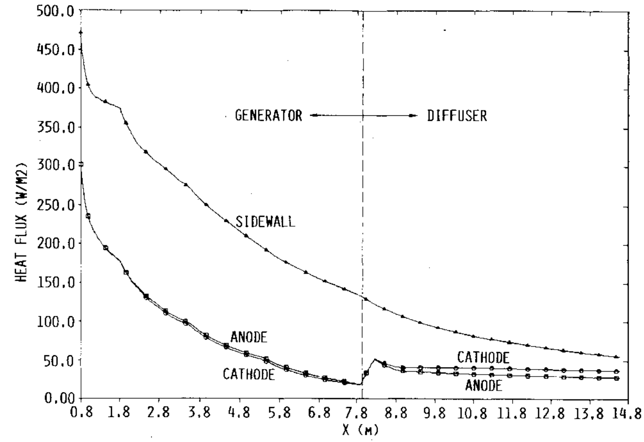


**Fig. 4 Heat-flux distributions on electrode and sidewalls for a typical high interaction MHD generator and diffuser.**

electrodes, the anode boundary layer is developing under a higher average pressure than that of the cathode as shown in Fig. 1. Since the pressure gradient in an MHD channel is in the main favorable, this means that the anode is experiencing a "less favorable" pressure gradient than the cathode. The result of this distribution of pressure and Lorentz force is that the anode will begin to exhibit a lower shear stress and higher shape factor than the cathode and will, in general, be more susceptible to separation.

Figures 2-4 are graphs of the calculated results for the electrode boundary-layer shape factors, the electrode and sidewall skin-friction coefficients, and the electrode and sidewall heat fluxes, respectively. The rather regular waviness apparent in these results is not an anomaly of the calculation but results directly from the geometry of the channel design which is constructed from several linear sections each having a different area variation with $x$. The effect of the Hall current induced pressure asymmetry on the electrode boundary layers is clearly illustrated in these figures. In the latter part of the active channel ($6.0 \text{ m} \leq x \leq 7.9 \text{ m}$), the shape factors differ by approximately 8% while the skin-friction coefficients differ by typically 20%. The unconventional behavior in the sidewall skin friction is due to the relative absence of current in the comparatively cold insulating sidewall boundary layers. This phenomenon is discussed fully in Ref. 14.

Of particular interest is the behavior of the boundary-layer parameters in the diffuser ($x \geq 7.9$ m). Over the last few centimeters of the active channel the load resistence is increasing until finally the channel is open-circuited at approximately 7.9 m. Over the interval the relative behavior of the boundary layers on the anode and cathode begins to reverse. Eventually, after a short run in the diffuser, the boundary layer on what was the cathode wall exhibits a higher shape factor and lower skin friction than that on the opposing wall. This is a reaction to the sudden relief of the transverse Lorentz force. This force was formerly directed toward the wall in the cathode boundary layer and away from the wall in the anode boundary layer. When the channel unloads, these forces are no longer present and the boundary layers are suddenly out of force balance as well as suddenly acted upon by an adverse pressure gradient (rather than the favorable gradient present in the active channel). These calculations indicate that for diffusers of practical length, the asymmetry will persist with the boundary layer on what was the cathode wall being considerably more susceptible to separation. Note also in Fig. 3 the return to conventional behavior of the sidewall skin friction as the axial Lorentz force is relieved. The somewhat more abruptly changing behavior in the distributions of the boundary-layer parameters at approximately 9.25 m is again a consequence of the geometry. At this point the electrode walls become parallel while the divergence half angle on the sidewalls is reduced from 2.0 to 1.5 deg. In addition, the electrode wall surface temperature drops significantly in this region due to a change in surface composition, hence the abrupt rise in the heat flux shown in Fig. 4.

## References


[1] Demetriades, S. T., "Momentum Transfer to Plasmas by Lorentz Forces," *Physico-Chemical Diagnostics of Plasmas,* Northwestern Univ. Press, Evanston, Ill., 1964, pp. 297-328.

[2] Norman, W., and Siler, L. G., "Experiments on a Magnetohydrodynamic Nozzle Accelerator in a Shock Tunnel," Arnold Engineering Development Center Tech. Rept. AEDC-TR-63-53, March 1969.

[3] Doss, E. D., Argyropoulos, G. S. and Demetriades, S. T., "Two-Dimensional Flow Inside MHD Ducts with Transverse Asymmetries," *AIAA Journal,* Vol. 13, May 1975, pp. 545-546.

[4] Argyropoulos, G. S., Demetriades, S. T., and Kendig, A. P., "Influence of Nonuniformities, Thermal Diffusion, Boundary Layers and Finite Reaction Rates in Nonequilibrium J × B Devices," AIAA Paper 67-715, Sept. 1967.

[5] Demetriades, S. T. and Argyropoulos, G. S., "Ohm's Law in Multicomponent Nonisothermal Plasmas with Temperature and Pressure Gradients," *Physics of Fluids,* Vol. 9, No. 11, Nov. 1966.

[6] Argyropoulos, G. S., Demetriades, S. T., and Lakner, K., "Compressible Turbulent Magnetohydrodynamic Boundary Layers," *Physics of Fluids,* Vol. 11, Dec. 1968, pp. 2559-2566.

[7] Demetriades, S. T., Argyropoulos, G. S., and Maxwell, C. D., "Progress in Analytical Modeling of MHD Power Generators," *Proceedings of 12th Symposium on Engineering Aspects of Magnetohydrodynamics,* Argonne National Lab., March 1972, pp. I.5.1-I.5.13.

[8] Demetriades, S. T., et al., "STD/MHD Code Descriptions for Computer Programs Utilized under Contract AC-01-79-ET15501," STD Research Corp. Rept. No. STDR-79-3, Jan. 1979.

[9] Demetriades, S. T., et al., "Numerical Solutions of the Coupled Three-Dimensional Magnetohydrodynamic Channel Problem," STD Research Corporation Final Rept. No. STD-75-001 prepared for the National Science Foundation under Contract NSF-C-727, Jan. 1975.

[10] Van Driest, E. R., "On Turbulent Flow Near a Wall," *Journal of the Aerospace Sciences,* Vol. 23, 1956, p. 1007.

[11] Markham, D. M., Maxwell, C. D., and Oliver, D. A., "A Numerical Solution to the Unsteady, Quasi-Three-Dimensional, Turbulent Heat Transfer Problem in an MHD Channel," ASME Paper 77-HT-90, Salt Lake City, Aug. 1977.

[12] Demetriades, S. T., Argyropoulos, G. S., Curry, B. P., and Patel, N. J., "Numerical Solutions of the Coupled Three-Dimensional Magnetohydrodynamic Channel Problem," STD Research Corp. Final Rept. No. STD-75-001 prepared for the National Science Foundation under Contract NSF-C-727, Jan. 1965.

[13] *Preliminary Proceedings of AFOSR-IFP-Stanford 1968 Conference on Turbulent Boundary Layers, Vol. II: Compiled Data,* Stanford, Calif., 1968.

[14] Maxwell, C. D., Demetriades, S. T., Oliver, D. A., Vetter, A. A., Swean, T. F., "Scale-Up of Advanced MHD Generators," AIAA Paper 80-0179, Pasadena, Calif., Jan. 1980.


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# Mixing Length in Low Reynolds Number Turbulent Boundary Layers


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## Nomenclature

| | |
|---|---|
| $G$ | = Clauser shape parameter, $\sqrt{(2/C_f)}\ (1-1/H)$ |
| $H$ | = shape parameter, $\delta^*/\theta$ |
| $K$ | = pressure gradient parameter, $(\nu/U_e^2 \mathrm{d}U_e/\mathrm{d}x)$ |
| $l$ | = mixing length |
| $M$ | = Mach number |
| $u_\tau$ | = friction velocity, $\tau_w/\rho_w$ |
| $\beta$ | = pressure gradient parameter, $(\delta^*/\tau_w \mathrm{d}p/\mathrm{d}x)$ |
| $\Delta_p$ | = pressure gradient parameter, $(\nu/\rho u_\tau^3 \mathrm{d}p/\mathrm{d}x)$ |
| $\delta$ | = velocity boundary-layer thickness where $U/U_e = 0.995$ |
| $\delta^+$ | = Reynolds number, $u_\tau \delta/\nu_w$ |
| $\delta^*$ | = boundary-layer displacement thickness |
| $\tau_w$ | = wall shear stress |
| $\nu$ | = viscosity |
| $\rho$ | = density |

*Subscripts*

| | |
|---|---|
| $m$ | = maximum value |
| 0.5 | = evaluated at $y/\delta = 0.5$ |
| $e$ | = edge |
| $w$ | = wall |

## Introduction

BUSHNELL et al.[1] analyzed existing data to show that, at low values of the Reynolds number $\delta^+$, the mixing-length parameter $(l/\delta)_m$ was smaller at large distances downstream of transition than at small distances, for given $\delta^+$. The data at small distances came from measurements on flat plates and other isolated bodies, whereas the measurements at large distances were made on the nozzle walls of supersonic wind tunnels. Comparison of Table 1 and Fig. 2 of Ref. 1 shows that nearly all the measurements which show a lower than usual mixing-length parameter were taken at Mach numbers above 5. Existing low-speed data (e.g., Refs. 2 and 3) indicate that the mixing-length parameter increases with decreasing Reynolds number on flat plates, so that the question of whether the reduction found by Bushnell et al. occurs only at high Mach numbers or whether it is a universal feature of low-Reynolds-number turbulent boundary layers remained open. This Note presents the results of an experimental and computational investigation of this question.[4]

## Low-Speed Experiment

### Experimental Arrangement

To produce a low Reynolds number $\delta^+$ far downstream of transition we set up, in the Imperial College 0.762 × 0.127 m (30 × 5 in.) boundary-layer tunnel,[5] a flow with a favorable pressure gradient sufficiently strong to keep the momentum-thickness Reynolds number at or below 1000 for a distance of about 50 boundary-layer thicknesses. Over this distance, the momentum thickness Reynolds number on a flat plate would approximately double. The initially laminar boundary layer was tripped by a spanwise wire sized to ensure rapid recovery from transition of the inner layer,[6] as demonstrated by the

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